Geometric Representation of Signals.

The essence of geometric representation of signals is to represent any set of M energy signals $\{s_i(t)\}$ as linear combinations of N orthonormal basis functions, where $N \leq M$.

ie to say; given a set of real valued energy signals $s_1(t)$, $s_2(t)$ --- $s_M(t)$ each of duration T seconds we write.

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t), \begin{cases} 0 \leq t \leq T. \\ i = 1,2,\ldots M. \end{cases} \quad \text{--- (1)}$$

where the co-efficients of the expansion are defined by

$$s_{ij} = \int_0^T s_i(t)\cdot\phi_j(t)\,dt \quad \begin{matrix} i=1,2,\ldots M. \\ j=1,2,\ldots N. \end{matrix} \quad \text{--- (2)}$$

The real-valued basis functions $\phi_1(t)$, $\phi_2(t)$ --- $\phi_N(t)$. are orthonormal, by which we mean.

$$\int_0^T \phi_i(t)\cdot\phi_j(t)\,dt = \delta_{ij} = \begin{cases} 1 \text{ if } i=j \\ 0 \text{ if } i\neq j. \end{cases} \quad \text{--- (3)}$$

where δ_{ij} is the Kronecker ~~data~~ delta.

The 1st cond^n of eq (3) states that each basis function is normalized to have unit energy. The second cond^n states that the basis functions $\phi_1(t)$, $\phi_2(t)$ -- $\phi_N(t)$

are orthogonal w.r.t. each other over the interval. $0 \leq t \leq T$.

The set of co-efficients $\{s_{ij}\}_{j=1}^{N}$ may naturally be viewed as an N-dimensional vector, denoted by s_i. The vector s_i, bears a one to one relationship with the transmitted signal $s_i(t)$:

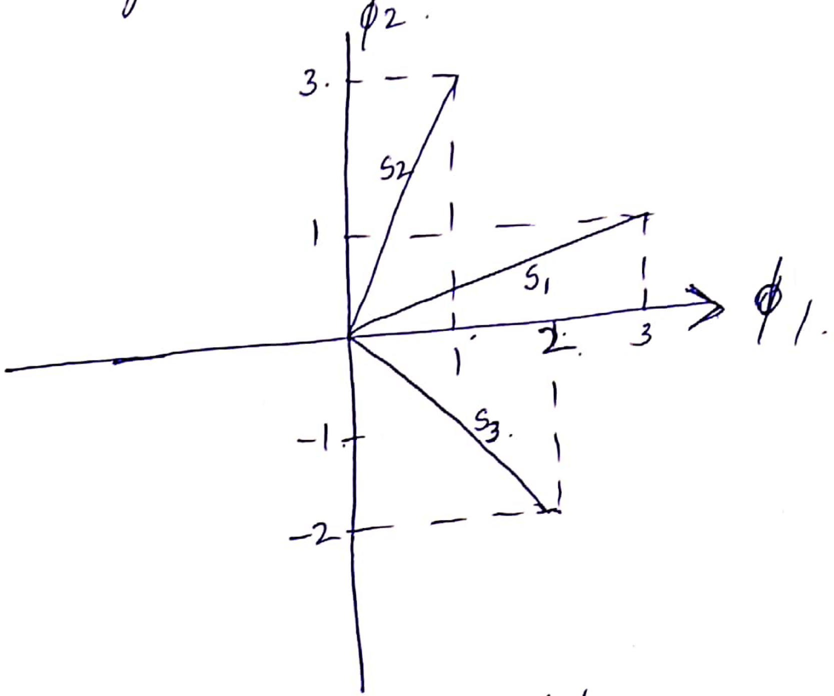
We may state that each signal in the set $\{s_i(t)\}$ is completely determined by the vector of its co-efficients.

$$s_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix} \quad i = 1, 2, \ldots M.$$

The vector s_i is called a signal vector, If we conceptually extend our conventional notion of 2 or 3 dimensional Euclidean spaces to an N-dimensional Euclidean space. We may visualize the set of signal vectors $\{s_i\}$ $i = 1, 2, 3, \ldots M$. as defining a corresponding set of M points in an N-dimensional Euclidean space, with N mutually perpendicular axes labeled $\phi_1, \phi_2, \ldots, \phi_N$. This N-dimensional Euclidean space is called the signal space.

The idea of visualizing a set of energy signals geometrically, ~~is~~ of profund importance... It provides the mathematical basis for the geometric representation of energy signals, thereby paving the way for the noise analysis of digital commn.

For the case of a 2 dimensional signal space with 3 signals, this is $N=2$, and $M=3$.



In an N-dimensional Euclidean space, we may define lengths of vectors and angles betn vectors. The length (also called the absolute value or norm) of a signal vector S_i by the symbol $\|S_i\|$. The squared length of any signal vector S_i is defined to be the inner product or dot product of S_i with itself as shown by.

$$\|S_i\|^2 = S_i^T S_i = \sum_{j=1}^{N} S_{ij}^2 \quad \text{where } i = 1, 2, 3, \ldots M.$$

where s_{ij} is the j element of s_i.

There is an interesting relationship bet^n the energy content of a signal and its representation as a vector. By definition, the energy of a signal $s_i(t)$ of duration 'T' sec is.

$$E_i = \int_0^T s_i^2(t)\, dt .$$

∴ Substituting eq ① in the above eqn.

$$E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\,\phi_j(t)\right]\left[\sum_{K=1}^{N} s_{iK}\,\phi_K(t)\right] dt .$$

Interchanging the order of summation and integration, we get

$$E_i = \sum_{j=1}^{N}\sum_{K=1}^{N} s_{ij}\cdot s_{iK} \int_0^T \phi_j(t)\,\phi_K(t)\, dt .$$

But since the $\phi_j(t)$ form an orthogonal set, in accordance with the 2 conditions of eq ③ we find that the above eqn reduces to.

$$E_i = \sum_{j=1}^{N} s_{ij}^2 = \|s_i\|^2 .$$

Thus the energy of a signal $s_i(t)$ is equal to the squared length of the signal vector $s_i(t)$.

In case of a pair of signals $s_i(t)$ and $s_k(t)$ represented by the signal vectors s_i and s_K respectively, we may also show that

$$\int_0^T s_i(t)\, s_k(t)\, dt = s_i^T . s_K.$$

Eq states that the inner product of the signals $s_i(t)$ and $s_K(t)$ over the interval $[0,T]$ using their time domain representations, is equal to the inner product of their respective vector representations s_i and s_K.

Note: The inner product of $s_i(t)$ and $s_K(t)$ is invariant to the choice of basis functions $\left(\phi_j(t)\right)_{j=1}^{N}$ in that it only depends on the components of the signals $s_i(t)$ and $s_K(t)$ projected onto each of the basis functions.

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Another useful relation involving the vector representation of the signal $s_i(t)$ and $s_k(t)$ is described by.

$$\|s_i - s_k\|^2 = \sum_{j=1}^{N} (s_{ij} - s_{kj})^2 = \int_0^T (s_i(t) - s_k(t))^2 dt .$$

where $\|s_i - s_k\|$ is the Euclidean distance d_{ik}, bet^n the points represented by the signal vectors s_i and s_k.

We now need to have a representation for the angle θ_{ik} subtended bet^n 2 signals s_i and s_k. By def^n, the ~~cosi~~ cosine of the angle θ_{ik} is equal to the inner product of these 2 vectors divided by the product of their individual norms.

$$\cos \theta_{ik} = \frac{s_i^T s_k}{\|s_i\| \, \|s_k\|} .$$

The 2 vectors s_i and s_k are thus orthogonal or $\perp$ to each other if their inner product $s_i^T s_k$ is zero, in which case $\theta_{ik} = 90°$, this cond^n is intuitively satisfying.

Synchronization :

For coherent reception, the receiver needs to be synchronous with the transmitter.

We say that the X'mitter & receiver are synchronous when the events in one sequence and the corresponding events in the other occur simultaneously.

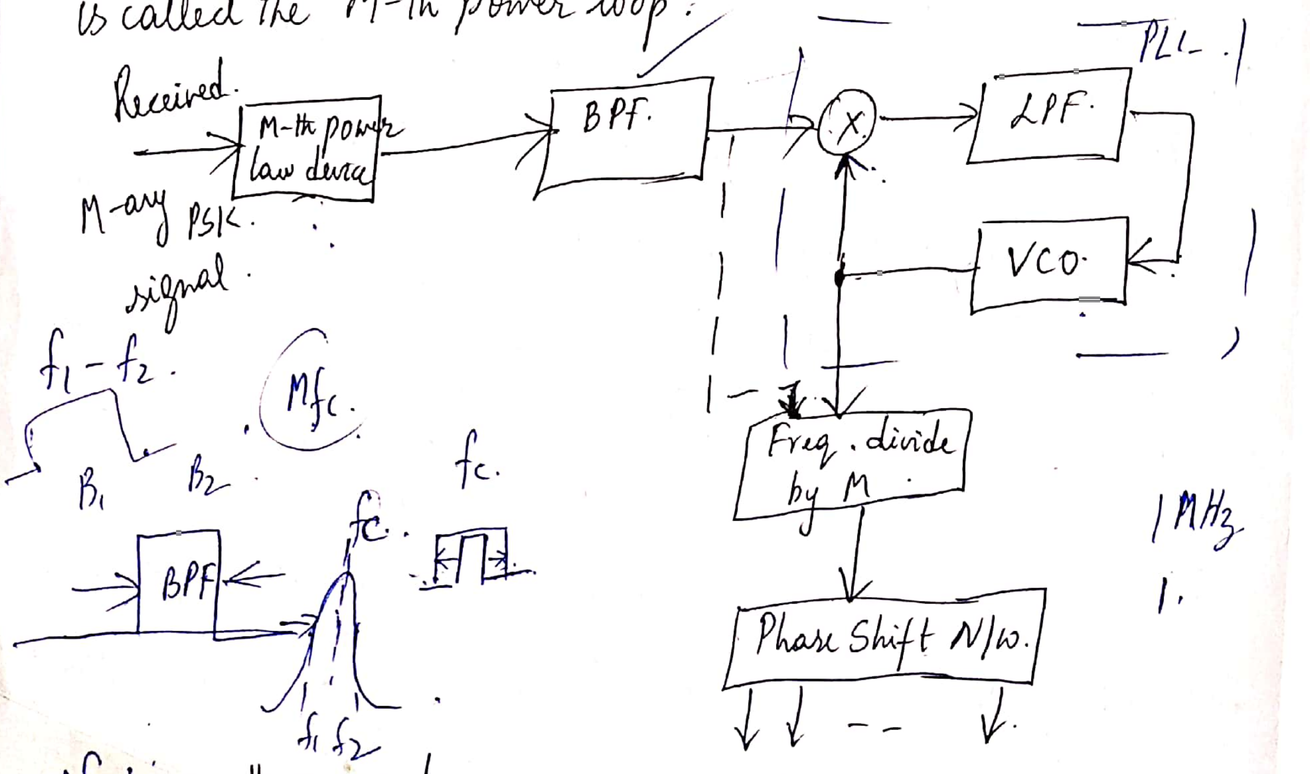
There is need for 2 basic modes of synchronization.

1. When coherent detection is used, knowledge of both freq & phase of the carrier is necessary. The estimation of carrier phase & freq is called as carrier synchronization.

2. The receiver also needs to know the starting & finishing times of the individual symbols, so that it may determine when to sample and when to quench the product integrators. The estimation of these times is called clock recovering or symbol synchronization.

Carrier Synchronization: The most straightforward method of carrier synchronization is to encertain that the power spectrum of the modulated signal contains a discrete component at the carrier freq. Then a narrow band PLL can be used to track this component, there by providing the desired reference signal at the receiver. A PLL contains at VCO, a loop filter and a multiplier that is connected in the form of a -ve f/b system.

Dis adv: The residual component does not convey any information other than freq & phase and hence represents a waste of power. Thus the receiver requires the use of a suppressed carrier tracking. Fig shows the block diagram of a carrier recovery ckt for M-ary PSK. This ckt is called the M-th power loop.



$\pm\Delta f_c +$ Mth power loop.

$f_c = 10\,Hz$

The received signal is passed through a BPF.

The signal is next passed through a N/w whose o/p is its input raised to the M^{th} power. At the o/p of the M-power device, there are many spectral components, one of which is a sinusoid of freq Mf_c. This M^{th} power o/p signal is then passed through a NBF to isolate the sinusoid at freq Mf_c and to remove more of the noise. Finally a divide by M. ckt yields the desired carrier of freq f_c.

The carrier freq generated at the x'mitter is not constant. It is subject to jitter ie variations. The Osc freq drifts back and forth through a freq deviation of $\pm \Delta f$. The nominal freq at which this drifting occurs be f_d.

Normally $\Delta f \gg f_d$.

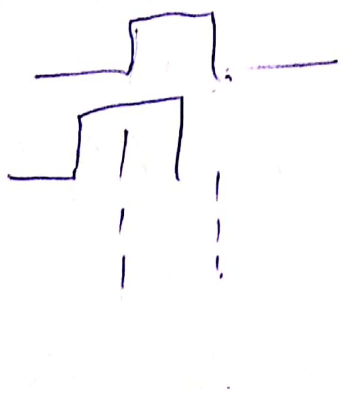
Typically. ~~$\Delta f = 10$ H~~ Δf is exceeding $\Delta f = 10\,Hz$. while rate of freq change might be less than 1 Hz.

Because of this the narrow band cannot be made as narrow as we would ~~like~~ like. Its BW must be ~~~~ some what larger than the width of $2M\Delta f$ to accomodate $\pm \Delta f$ multiplied by M. due to the M-power device..

Now if the NBF is replaced by a PLL. The PLL is used as a filter whose band pass is determined by the low pass filter. If the LPF has a trans function

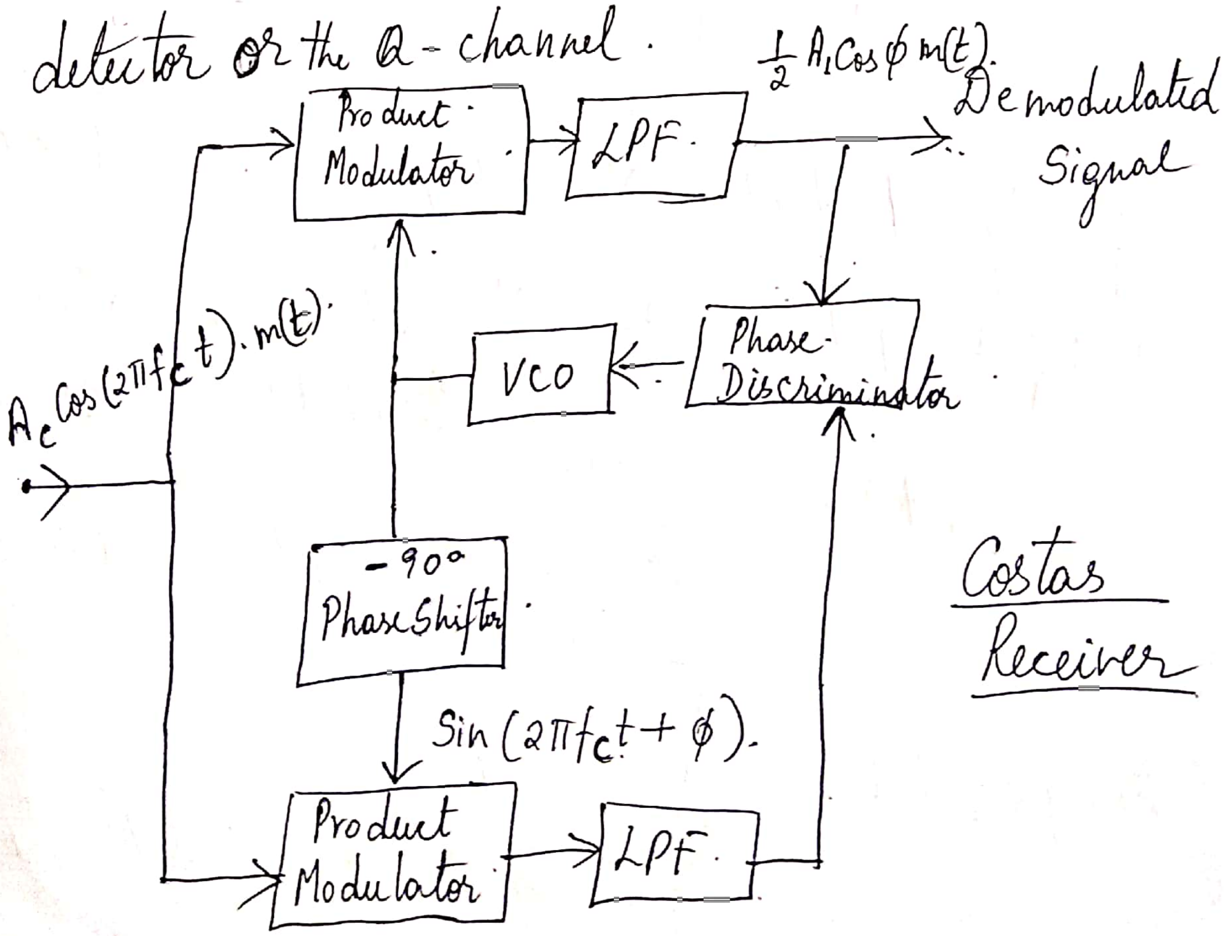
$$H(s) = \alpha\left(1 + \frac{1}{s}\right) = \alpha\left(\frac{s+1}{s}\right)$$

& if the PLL is adjusted to have a pass band $2Mf_d$ (rather than $2M\Delta f$), the PLL will still be able to follow the oscillator jitter. The PLL filter will have the reduced pass band of $2Mf_d$ and therefore the noise power too will be reduced.



The Costas Loop:

This ckt involves 2 PLL's employing a common VCO and loop filter. Assume that initially the VCO is operating at the carrier freq. (angular freq of ω_c). The incoming signal is fed to both the coherent detectors which is $A_c \cos(2\pi f_c t).m(t)$. The detector in the upper path is referred to as the in-phase coherent detector or I-channel and that in the lower path is the quadrature-phase coherent detector or the Q-channel.



Costas Receiver

The local oscillator signal is of the same phase as the carrier wave ie $A_c \cos(2\pi f_c t)$ used to generate the modulated wave. Under these conditions, ~~we~~ I-channel o/p contains the desired demodulated signal m(t) whereas the Q-channel o/p is zero due to quadrature null-effect of the Q-channel.

If the local oscillator phase drifts from its proper value by a small angle ϕ radians. The I-channel o/p will remain essentially unchanged, but there will now be some signal appearing at the Q-channel o/p which is proportional to $\sin\phi \simeq \phi$ for small ϕ. This Q-channel o/p will have the same polarity as the I-channel o/p for one direction of local osc phase drift and opposite polarity for the opposite direction of LC phase drift. Thus by combining the I & Q-channel o/p's in a phase discriminator, a DC control signal is obtained that automatically corrects for local phase errors in the VCO.

Detection of Non-coherent signals:

Structure of Non-coherent receivers.

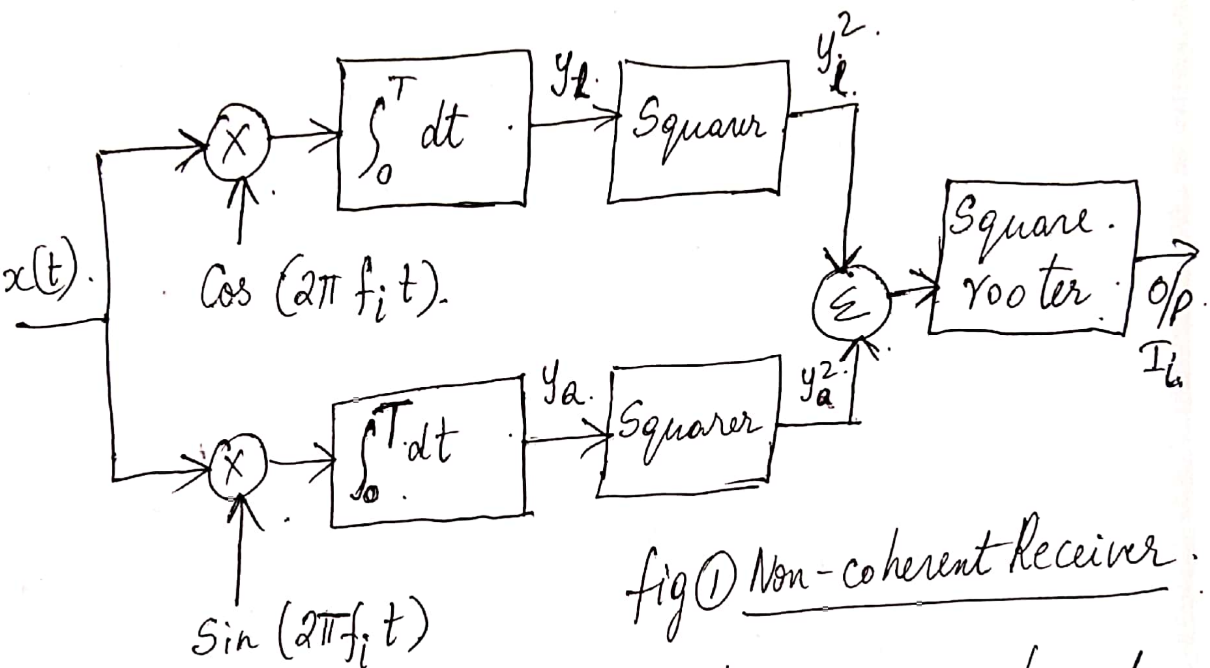


fig ① Non-coherent Receiver.

Based on the equations related to non-coherent receivers, the above structure is the general structure of non-coherent receivers and is known as the quadratic receiver. A digital commn. receiver with no provision made for carrier phase recovery is said to be non-coherent.

Two equivalent forms of the quadratic receiver are given below, the 1st form is obtained easily by replacing each correlator in fig ① with a corresponding equivalent matched filter.

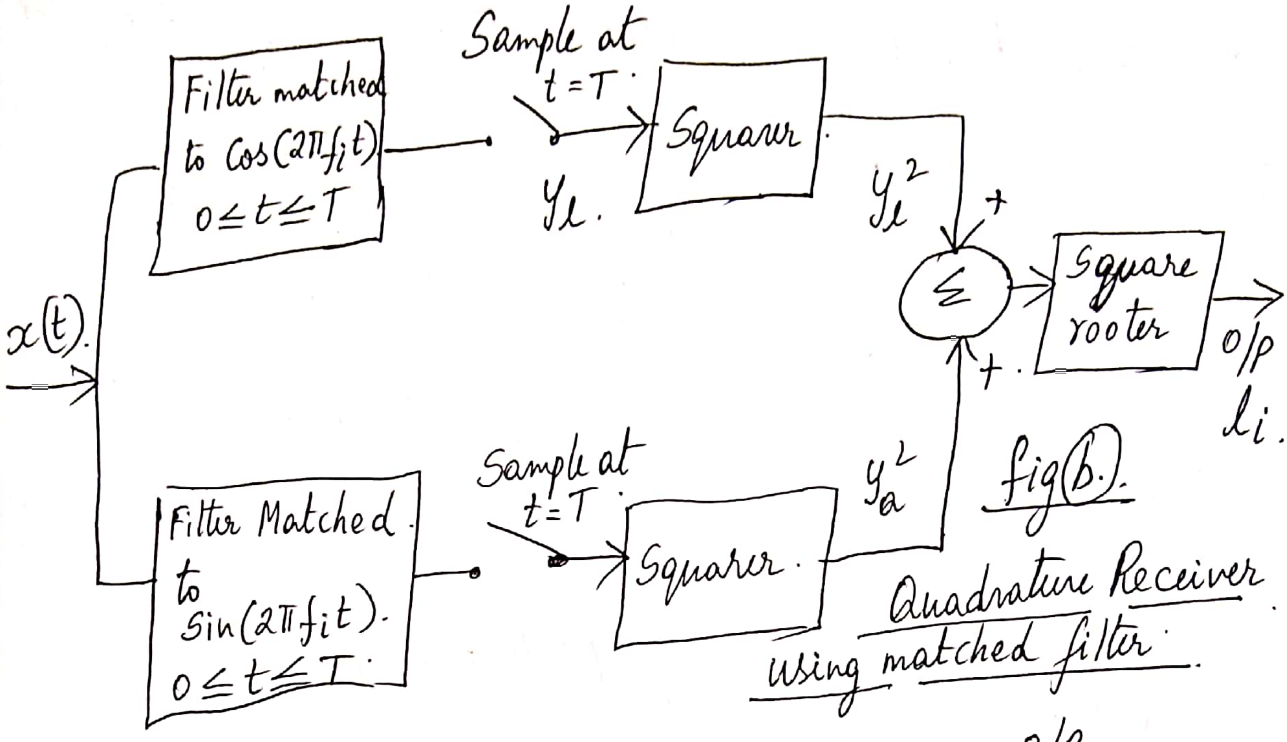


fig (b). Quadrature Receiver using matched filter.

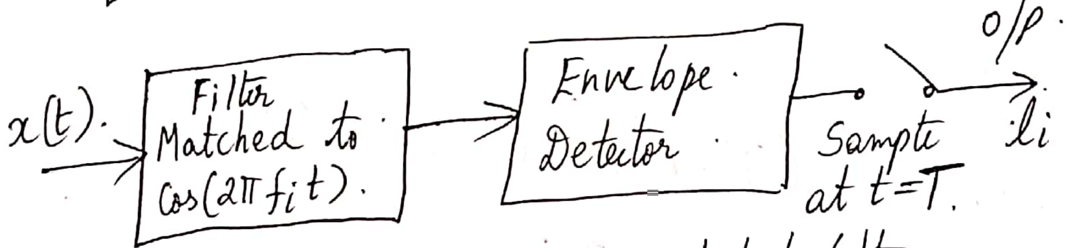


fig (c). Non-coherent matched filter.

Thus the alternative form of quadrature receiver shown in fig (b) is achieved. In one branch, there is a filter matched to the signal $\cos(2\pi f_i t)$ and in the other a filter matched to $\sin(2\pi f_i t)$. The filter o/p's are sampled at time $t = T$ squared and then added together.

To obtain the second equivalent form of the quadrature receiver, consider a filter that is matched to $s(t) = \cos(2\pi f_i t + \theta)$ for $0 \le t \le T$. The envelope.

of the matched filter o/p is obviously unaffected by the value of phase θ.

Choose a matched filter with impulse response $\cos(2\pi f_i(T-t))$, corresponding to $\theta=0$. The o/p of such a filter in response to the received signal $x(t)$ is given by.

$$y(t) = \int_0^T x(\tau)\cos[2\pi f_i(T-t+\tau)]\,d\tau.$$

$$= \cos[2\pi f_i(T-t)]\int_0^T x(\tau)\cos(2\pi f_i\tau)\,d\tau \quad -(1)$$

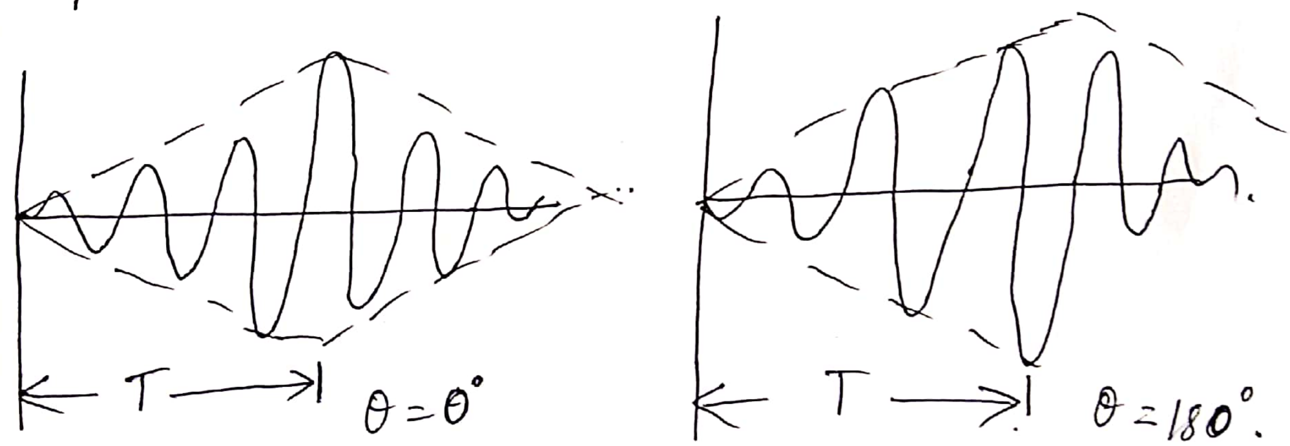
$$-\sin(2\pi f_i(T-t))\int_0^T x(\tau).\sin(2\pi f_i\tau)\,d\tau.$$

The envelope of the matched filter o/p is proportional to the square root of the sum of the squares of the integrals in the above eq (1). The envelope, evaluated at time $t=T$ is therefore.

$$l_i = \left\{\left[\int_0^T x(\tau)\cos(2\pi f_i\tau)\,d\tau\right]^2 + \left[\int_0^T x(\tau)\sin(2\pi f_i\tau)\,d\tau\right]^2\right\}^{1/2}.$$

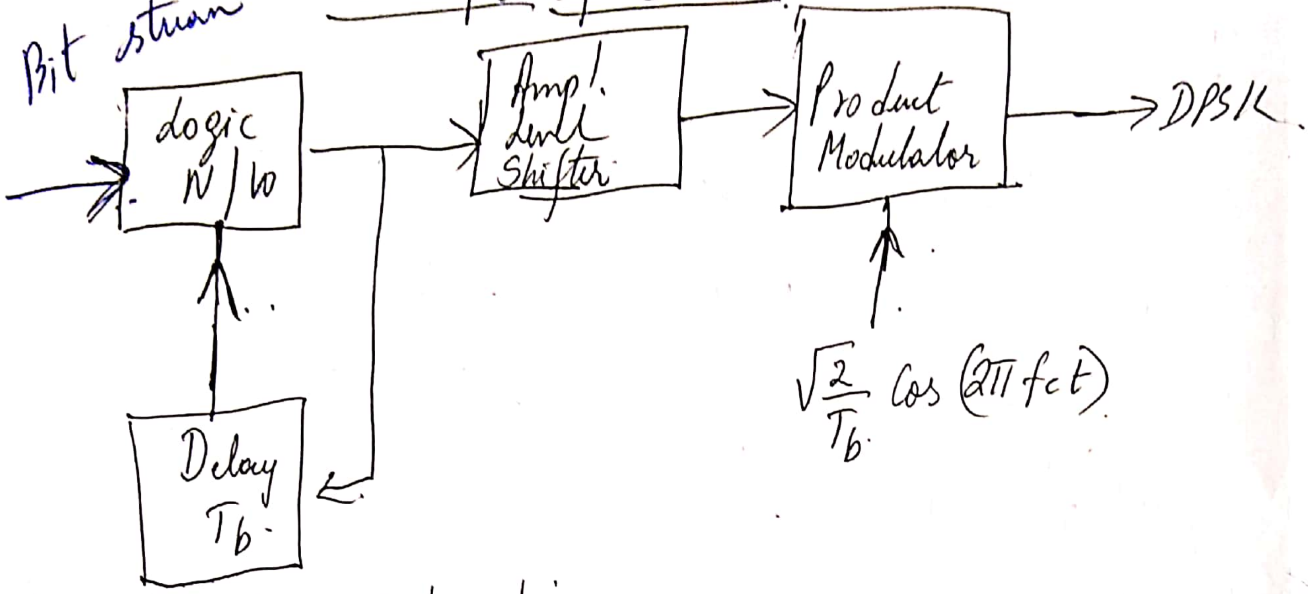
Therefore the o/p (at time T) of a matched filter — (2). to the signal $\cos(2\pi f_i t+\theta)$, of arbitary phase θ, followed by an evelope detector is the same as the corresponding o/p of the quadrature receiver. The combination of matched filter and envelope detector shown in fig 'c' is called a non-coherent matched filter.

The o/p of a filter matched to a rectangular RF wave reaches a +ve peak at the sampling instant t=T, however, the phase of the filter is not matched to that of the signal, the peak may occur at a time different from the sampling instant. Fig illustrates the filter o/p for θ=0° & θ=180°. It is seen that retaining only the envelope is best when prior information about the phase θ is not present.



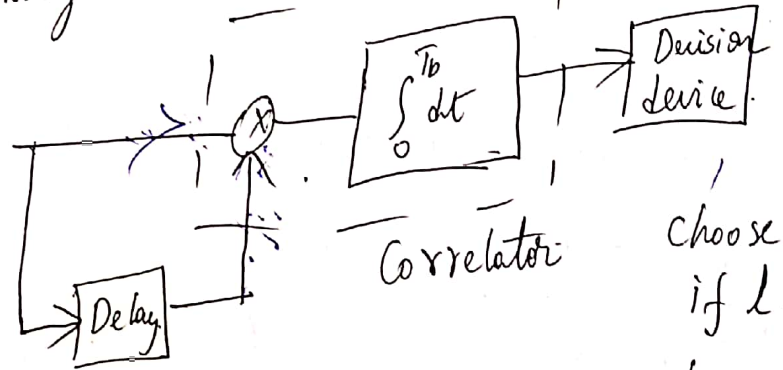
o/p of matched filter for a rectangular RF wave.

Principle of DPSK.



Symbol '0' 180° phase shift
'1' - unchanged

$x(t)$



Choose 1 if $l > 0$.
Choose 0 if $l < 0$.

Symbol 1 at the X'mitter i/p for the second part of this interval $T_b \le t \le 2T_b$.

$$s_1(t) = \begin{cases} \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) & 0 \le t \le T_b \\ \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) & T_b \le t \le 2T_b \end{cases}$$

phase

Binary symbol 0 at the X'mitter i/p.

$$s_2(t) = \begin{cases} \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) & 0 \le t \le T_b \\ \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t + \pi) & T_b \le t \le 2T_b \end{cases}$$